Math 53: Multivariable Calculus **Sections 201, 204**

Worksheet answers for 2021-10-04

If you would like clarification on any problems, feel free to ask me in person. (Do let me know if you catch any mistakes!)

Answers to conceptual questions

Question 1. As mentioned in class: if a region can be described algebraically using continuous functions and $=, \leq, \geq$ (but not $>$ or $<$) then it is closed. "Bounded" means that it doesn't go off to infinity in any direction, or in other words it is possible to enclose the entire region in a sufficiently large box.

(a) Closed
(b) Closed and bounded
(c) Closed and bounded
(d) Closed
(e) Bounded
(f) Closed

Question 2. This system of equations is the Lagrange multipliers system for the objective function $f(x, y) = e^{xy}$ on the constraint region $x^4 + y^6 = 2$. As mentioned in the previous problem, this is closed and bounded. Since f is continuous, EVT implies that it attains a max and min on this region, which are then necessarily solutions to the system of equations (with appropriate values of λ). Note that if the max and min are attained at the same point, then f is constant on the region, which means *every* point would be a solution. So in any case there are at least two solutions.

Question 3. You will find no solutions. However, there is an absolute minimum, namely at $(0, 0)$. The issue is that $g(x, y) = y^2 - x^3$ has zero gradient at that point, which is part of the constraint curve $g(x, y) = 0$; this manifests as a sharp "corner" of the curve.

Answers to computations

Problem 1. The function to maximize/minimize is just $f(x, y, z) = z$. So our system of equations is

$$\begin{aligned} 0 &= 4\lambda + 2x\mu \\ 0 &= -3\lambda + 2y\mu \\ 1 &= 8\lambda - 2z\mu \\ 4x - 3y + 8z &= 5 \\ z^2 &= x^2 + y^2. \end{aligned}$$

The first two equations give $(6x + 8y)\mu = 0$, so either $\mu = 0$ or $3x + 4y = 0$. The former results in no solutions, as either of the first two equations implies $\lambda = 0$, but then the third equation is not satisfied. Hence we are left with the second case, and the system of equations

$$\begin{aligned} 3x + 4y &= 0 \\ 4x - 3y + 8z &= 5 \\ z^2 &= x^2 + y^2. \end{aligned}$$

After some algebra, we find the solutions $(x, y, z) = (-4/3, 1, 5/3)$ and $(4/13, -3/13, 5/13)$. The former is the highest point and the latter is the lowest point, because $5/3 > 5/13$.